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New Cosmology

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We propose a model of our universe as a 3-sphere resting on the surface of a black hole which exists in a spacetime consisting of four space dimensions and one time dimension. The matter and energy within our universe exist as stationary solutions to the field equations in the Rindler coordinates just above the horizon of the black hole. Each solution may be thought of as a standing wave consisting of a wave propagating toward the horizon superposed with its time-reversed twin propagating away from the horizon. As matter and energy from the greater five-dimensional spacetime fall into the black hole, its radius increases and our universe expands. This mechanism of expansion allows the model to describe a universe which is older than its oldest stars and homogeneous without inflation. It also predicts galaxy counts at high redshift which agree with observation.

1. INTRODUCTION

The most popular model of the universe, that variation of the Big Bang model known as the Einstein–de Sitter model (Einstein and de Sitter, 1932), has been shown to disagree with recent astronomical observations. The number of galaxies per unit redshift at a redshift of 0.25 has been estimated from observation to be three to four times higher than that predicted by the model (Lilly *et al.*, 1991). Estimates of the age of the universe obtained from the model and recent measurements of the Hubble constant yield a value of about eight billion years (Kennicutt *et al.*, 1995), while estimates of the ages of the oldest stars turn out to be about 16 billion years (Demarque *et al.*, 1991). It is impossible to make a definite statement at present, but if this trend continues the Big Bang model will eventually run into serious difficulty.

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In a previous paper (McFarland, 1997) we proposed a model of spacetime which was 5-dimensional and Rindler in nature. Its line element was

$$ds^2 = -\frac{z^2}{L^2}\, dt^2 + dz^2 + dx^2 \tag{1.1}$$

where L is a parameter with units of length and $dx^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$. The coordinates x^1, x^2, and x^3 are those of the 3-dimensional space we live in, while z is a Rindler coordinate. Stationary solutions to the field equations in this spacetime were obtained for scalar, spinor, and electromagnetic fields. The stationary nature of these solutions indicated that the probability of finding the associated particle in the space above the Rindler horizon was constant in time. The solutions seem to hang motionless above the horizon in spite of the gravitational field pulling them toward it.

The solutions were all found to attenuate as $\exp(-pz)$, where p is the momentum of the associated particle. This confines the solutions to a narrow region in z and, assuming these solutions to comprise all the matter and energy we can observe, provides us with the illusion that our space is 3-dimensional. This confinement may be viewed as the trapping of the associated particles in the gravity well of the gravitational field which points toward the Rindler horizon.

We showed that, although the field equations contain no mass term, solutions exist for which the energy E and momentum p of the associated particle satisfy $E > p$. The model thus permits the existence of mass without spoiling gauge symmetry and without the Higgs mechanism (Higgs, 1964). This was our motivation for introducing it.

The model, although sufficiently motivated, was not very compelling because the Rindler nature of our spacetime was introduced as an *a priori* assumption. No convincing mechanism responsible for it was given. In this paper we provide such a mechanism. It will lead us to a cosmological model in which the universe expands as is the case with the Einstein–de Sitter model. The mechanism behind the expansion, however, is dramatically different in the two models. The model we propose has no initial Big Bang. It describes a universe which is much older than that of the Einstein–de Sitter model and is thus compatible with estimates of the ages of the oldest stars. It yields the correct number of galaxies/unit redshift and, in an unforced and natural way, it can also explain the large-scale homogeneity of the universe without inflation (Guth, 1981).

2. THE MODEL

We begin by assuming the universe to exist within a five-dimensional spacetime consisting of four space dimensions and one time dimension.

Hereafter this spacetime will be referred to as the 4D universe. The universe we live in has, of course, three space dimensions and one time dimension. Hereafter it will be referred to as the 3D universe. It will be seen to arise naturally in this model. We assume that Einstein's field equations in vacuum are true in the 4D universe. With the Ricci tensor given by

$$R_{\mu\nu} = \partial_\nu \Gamma^\lambda_{\mu\lambda} - \partial_\lambda \Gamma^\lambda_{\mu\nu} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho} - \Gamma^\rho_{\mu\nu}\Gamma^\lambda_{\lambda\rho} \tag{2.1}$$

these field equations read

$$R_{\mu\nu} = 0 \tag{2.2}$$

The $\Gamma^\lambda_{\mu\nu}$ are the Christoffel symbols,

$$\Gamma^\lambda_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \tag{2.3}$$

and all indices run from 0 through 4. The coordinates x^μ are defined such that x^0 is time and x^μ, $\mu \neq 0$, is a space coordinate.

We will now obtain a spherically symmetric solution to equation (2.2) in the 4D universe. This solution is the analog of the Schwarzschild solution in the 3D universe.

We first assume the metric tensor to have the following spherically symmetric five-dimensional form.

$$\begin{gathered} g_{00} = -B(r), \qquad g_{rr} = A(r), \qquad g_{\chi\chi} = r^2 \\ g_{\theta\theta} = r^2 \sin^2\chi, \qquad g_{\phi\phi} = r^2 \sin^2\chi \sin^2\theta \end{gathered} \tag{2.4}$$

with all other $g_{\mu\nu} = 0$ and $g^{\mu\nu} = (g_{\mu\nu})^{-1}$ for $\mu = \nu$. In equation (2.4), r is the radial coordinate and χ, θ, and ϕ are the three angular coordinates of four-dimensional space.

Inserting these $g_{\mu\nu}$ into equation (2.3) yields values for the 125 Christoffel symbols. Of these, 22 are nonzero and are listed in the appendix. Using these in equation (2.1) yields the following relevant Ricci tensor components:

$$\begin{aligned} R_{00} &= -\frac{B''}{2A} + \frac{1}{4}\left(\frac{B'}{A}\right)\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{3}{2r}\frac{B'}{A} \\ R_{rr} &= \frac{B''}{2B} - \frac{1}{4}\left(\frac{B'}{B}\right)\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{3}{2r}\frac{A'}{A} \\ R_{\chi\chi} &= -2 + \frac{r}{2A}\left(-\frac{A'}{A} + \frac{B'}{B}\right) + \frac{2}{A} \end{aligned} \tag{2.5}$$

where a prime denotes the operation d/dr. These equations and equation (2.2) then yield

$$0 = \frac{R_{rr}}{A} + \frac{R_{00}}{B} = -\frac{3}{2rA}\left(\frac{A'}{A} + \frac{B'}{B}\right)$$

and so

$$A(r) = \frac{1}{B(r)} \tag{2.6}$$

Equations (2.5) and (2.6) yield

$$rB' + 2B = 2$$

with solution

$$B(r) = 1 + \frac{k}{r^2}$$

where k is a constant. In four-dimensional space the gravitational field of mass M at large $\mathbf{r}$ is $\mathbf{g} = -GM\mathbf{r}/r^4$, where G is the analog of the Newtonian gravitational constant in four-dimensional space. General relativity requires $\mathbf{g} = \frac{1}{2}\nabla g_{00} = -\frac{1}{2}\nabla B(r)$. This then gives $k = GM$ and so

$$B(r) = 1 - \frac{GM}{r^2}, \qquad A(r) = \left(1 - \frac{GM}{r^2}\right)^{-1} \tag{2.7}$$

Equations (2.4) and (2.7) then give us the following five-dimensional line element:

$$ds^2 = -\left(1 - \frac{GM}{r^2}\right) dt^2 + \left(1 - \frac{GM}{r^2}\right)^{-1} dr^2 + r^2\, d\chi^2 + r^2 \sin^2\chi\, d\theta^2 + r^2 \sin^2\chi \sin^2\theta\, d\phi^2 \tag{2.8}$$

An obvious consequence of equation (2.8) is the existence of black holes in the 4D universe. A black hole of mass M would have a radius R which satisfies

$$R^2 = GM \tag{2.9}$$

Another consequence of equation (2.8) can be seen by making the following coordinate transformation:

$$t' = \left(\frac{L^2}{GM}\right)^{1/2} t$$

$$z = (GM)^{1/2}\left(1 - \frac{GM}{r^2}\right)^{1/2}$$

$$x^1 = (GM)^{1/2}\left(\chi - \frac{\pi}{2}\right)$$

$$x^2 = (GM)^{1/2}\left(\theta - \frac{\pi}{2}\right)$$

$$x^3 = (GM)^{1/2}\phi$$

Under this transformation and near the point $(r, \chi, \theta, \phi) = ((GM)^{1/2}, \pi/2, \pi/2, \phi)$, equation (2.8) becomes

$$ds^2 = -\frac{z^2}{L^2}(dt')^2 + dz^2 + dx^2 \tag{2.10}$$

where $dx^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$. Equations (2.10) and (1.1) are identical. Rindler coordinates may therefore be used to describe spacetime near the horizon of a black hole in the 4D universe. This fact, together with the existence of the stationary solutions in Rindler coordinates described in our previous paper, allow us to propose the following cosmological model.

The model can summarized as follows: Our 3D universe rests upon the surface of a four-dimensional black hole.

This black hole formed within the greater 4D universe at some time in the past. It probably formed much in the way an ordinary three-dimensional black hole might form at the center of a galaxy, by accumulation of material in a region of space through the action of gravity. The material that formed the four-dimensional black hole was drawn from distant regions of the 4D universe and most of it now resides within the horizon of the four-dimensional black hole. Some of it, however, never made it that far. Some of the material became trapped as stationary solutions to the field equations in the Rindler spacetime just above the horizon of the black hole. It remains there today and comprises all of the matter and energy in our 3D universe.

Like any three-dimensional black hole, the four-dimensional black hole will capture and swallow anything on a suitable trajectory. We assume the 4D universe to contain material which today is still falling into the four-dimensional black hole. As this material falls in, the radius of the four-dimensional black hole increases. Our 3D universe, riding on the surface of this black hole, thus expands. This expansion depends upon the distribution of material in the 4D universe and is clearly unrelated to the amount of matter in our 3D universe. According to this model, then, there is no point in trying to settle the question of whether or not the universe is closed by looking around to see if there is enough matter to close it. The model says that our 3D universe is closed, but that the matter within it has nothing to do with its closure. Our 3D universe owes its closure to the fact that it is a

3-sphere resting upon the surface of a four-dimensional black hole and this fact owes its existence to mass within the horizon of the four-dimensional black hole.

The four-dimensional black hole will continue to expand as long as material falls into it. As far as we know, this process will continue forever. Our 3D universe, although closed, may thus expand forever. As to how long it has been expanding, we cannot say. The four-dimensional black hole could have formed at any time in the past (subject to the constraint that it has existed long enough for the light from distant galaxies to have reached us).

Another important feature of this model is that it has no need of a Big Bang. The four-dimensional black hole probably formed quietly. It could, however, have formed with the contents of our 3D universe at a high enough temperature for matter and radiation to exist in equilibrium. The remnant today of this would be the cosmic microwave background radiation. In the next section, we assume that matter and radiation were once coupled and estimate how long ago the decoupling occurred.

3. TIMESCSALE

In the last section, we showed that as material falls into the four-dimensional black hole, our 3D universe expands. The rate at which the mass M of the black hole increases is equal to the rate at which energy falls into it, which in turn must be proportional to the surface volume the black hole exposes to the 4D universe. This surface volume is proportional to R^3, where R is the radius of the four-dimensional black hole (and of our 3D universe). We therefore have

$$\frac{dM}{dt} = A'R^3 \tag{3.1}$$

where A' is a constant and t is time.

Equations (2.9) and (3.1) yield

$$\frac{dR}{dt} = AR^2 \tag{3.2}$$

where $A = GA'/2$. Equation (3.2) has the solution

$$t = \frac{1}{A}\left(\frac{1}{R_0} - \frac{1}{R}\right) \tag{3.3}$$

where t is the time for our 3D universe to expand from radius R_0 to radius R.

With equation (3.2) we can write the Hubble constant H as

$$H = \frac{1}{R}\frac{dR}{dt} = AR \tag{3.4}$$

which can be used to write equation (3.3) in the following more convenient form:

$$t = \frac{1}{H}\left(\frac{R}{R_0} - 1\right) \tag{3.5}$$

Matter decoupled from radiation at a redshift of $z = 1400$ (Peebles, 1993). If the radius of our 3D universe then was R_0 and its radius is now R, then $R/R_0 - 1 = z = 1400$. With $H = 2.5 \times 10^{-18}\ s^{-1}$ (approximately 75 km/sMpc), equation (3.5) then yields $t = 1.77 \times 10^{13}$ years, a time which is much greater than the ages of the oldest observed stars.

The deceleration parameter $q = -\ddot{R}R/\dot{R}^2$ is found from equation (3.2) to have the value -2.

4. HOMOGENEITY

The cosmological model outlined here is compatible with the large-scale homogeneity of our 3D universe. To see this, we note that light can travel a distance $R\,d\chi$ in time dt, where $R\,d\chi/dt = 1$ and so $d\chi = dt/R$. From equation (3.3), R increases by dR during dt, where $dt = dR/AR^2$. Light therefore travels through $d\chi$ while R increases by dR, where

$$d\chi = \frac{dR}{AR^3}$$

Upon integrating we obtain

$$\chi = \frac{1}{2A}\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right) \tag{4.1}$$

as the angle through which light travels as R increases from R_1 to R_2.

Assume that today our 3D universe is homogeneous out as far as $\chi = \chi_H$ from our location. This means that at some time t_H in its early history, when $R = R_H$, all the matter out to $\chi = \chi_H$ was causally connected. Light therefore must have had a chance to travel through angle χ_H by the time

$R = R_H$. In the new cosmological model, this is possible if the 3D universe existed at some time $t_0 < t_H$ when $R = R_0$ such that

$$\chi_H = \frac{1}{2A}\left(\frac{1}{R_0^2} - \frac{1}{R_H^2}\right)$$

For any R_H and χ_H we might choose, we can find an R_0 which satisfies this equation.

5. TESTS

We now present two observational tests of the new cosmological model. The first test is a prediction of the measured flux I of electromagnetic energy from an object of known power output P at redshift z.

If the radius of our 3D universe was R_0 when the object emitted the light and R when we received the light, then the energy/photon and the photon arrival rate will both be reduced by factor R_0/R during the photons' flight. This reduces the flux by factor $(R_0/R)^2$. If the angle between us and the object is χ, then the area of a shell on which we reside which has the object at its center is $4\pi R^2 \sin^2\chi$. The flux we measure must then be

$$I = \left(\frac{R_0}{R}\right)^2 \frac{P}{4\pi R^2 \sin^2\chi} \tag{5.1}$$

In time dt, light travels through angle $d\chi = dt/R$. From equation (3.3), we obtain $R = R_0/(1 - AR_0t)$ and so

$$d\chi = \frac{1}{R_0}(1 - AR_0t)\, dt$$

In time t, then, light travels through angle χ, where

$$\chi = \frac{1}{R_0}\left(t - \frac{1}{2}AR_0t^2\right) \tag{5.2}$$

From equation (3.3) and the relation $z = R/R_0 - 1$, where z is the redshift, we have $t = z/AR$. Substituting this into equation (5.2) yields

$$\chi = \frac{z}{AR^2}\left(1 + \frac{1}{2}z\right) = \frac{z}{HR}\left(1 + \frac{1}{2}z\right) \tag{5.3}$$

where we have used equation (3.4).

Equations (5.1) and (5.3) yield our first prediction:

$$I = \frac{P}{4\pi(z + 1)^2 R^2 \sin^2((cz/HR)(1 + z/2))} \tag{5.4}$$

where we have explicitly displayed the factor c.

When we look at galaxies with redshift z, we see them when their density was ρ_0 and the radius of the universe was R_0. The number dN that we see in a shell centered at our location and of thickness $R_0\, d\chi$ is therefore $dN = 4\pi R_0^2 \rho_0 \sin^2\chi\, (R_0 d\chi)$, and so

$$\frac{dN}{d\chi} = 4\pi R_0^3 \rho_0 \sin^2\chi \tag{5.5}$$

From equation (5.3) we obtain $dz = HR\, d\chi/(z + 1) = HR_0\, d\chi$. We therefore have

$$\frac{dN}{dz} = \frac{1}{HR_0} \frac{dN}{d\chi} \tag{5.6}$$

Equations (5.3), (5.5), and (5.6) yield our second prediction:

$$\frac{dN}{dz} = \frac{4\pi c\rho}{H}(z + 1)R^2 \sin^2\left(\frac{cz(1 + z/2)}{HR}\right) \tag{5.7}$$

where $R_0 = R/(z + 1)$ and the present density ρ of galaxies satisfies $\rho_0 = \rho(z + 1)^3$. In equation (5.7), we have once again explicitly displayed factors of c.

A derivation similar to the one leading to equation (5.7), but performed within the Einstein–de Sitter model, yields (Peebles, 1993)

$$\frac{dN}{dz} = \frac{16\pi c^3 \rho}{H^3}(z + 1)^{-3/2}[1 - (z + 1)^{-1/2}]^2 \tag{5.8}$$

Evaluation of equations (5.7) and (5.8) at $z = 0.25$ with $R \to \infty$ in equation (5.7) reveals that at this redshift, the model presented here predicts a value of dN/dz which is 3.1 times larger than the one predicted by the Einstein–de Sitter model. The new model thus agrees much more closely with observation.

6. DISCUSSION

The cosmological model outlined here is free of most of the problems that appear to be associated with the Big Bang model. It describes a universe which is older than the stars it contains, and it predicts galaxy counts at high redshift which appear to coincide with observation. It also solves two problems which have allegedly already been solved, those of homogeneity and the gauge-invariant assignment of masses to elementary particles (McFarland, 1997).

The model's success with the problem of age, galaxy counts, and homogeneity can be traced to the fact that it describes a universe which expanded very slowly in its early history. The negative value of the deceleration parameter indicates that the expansion rate is actually increasing with time. The Big Bang model, on the other hand, describes a universe whose expansion rate was much greater in its early history and which is decreasing with time.

The initially slow expansion rate associated with the new model creates a problem which does not exist within the Big Bang model. The Big Bang model correctly predicts the abundance of helium which was created in the early history of the universe (Peebles, 1993). This abundance is sensitive to the rate at which the universe was expanding at the time. We have not estimated the primordial production of helium within the framework of the new model. It seems clear, however, since the universe expands much more slowly in the new model, that the new model would predict a helium abundance which differs from the one predicted by the Big Bang model.

It is possible, however, to wiggle out of this difficulty. At the time the four-dimensional black hole formed, the density of material in the region immediately external to the black hole was probably much higher than it is today. The constant A' in equation (3.1) would then be larger than it is today, and as a result our 3D universe would expand faster than expected in its early history. This early expansion rate could easily be adjusted to fit the observed helium abundance.

APPENDIX

Nonzero Christoffel symbols from the metric of equation (2.4) are

$$\Gamma^r_{rr} = \frac{1}{2A}\frac{dA}{dr}, \qquad \Gamma^r_{\chi\chi} = -\frac{r}{A}, \qquad \Gamma^r_{\theta\theta} = -\frac{r\sin^2\chi}{A}$$

$$\Gamma^r_{\phi\phi} = -\frac{r\sin^2\chi\sin^2\theta}{A}, \qquad \Gamma^r_{00} = \frac{1}{2A}\frac{dB}{dr}$$

$$\Gamma^\chi_{r\chi} = \Gamma^\chi_{\chi r} = \frac{1}{r}, \qquad \Gamma^\chi_{\theta\theta} = -\sin\chi\cos\chi, \qquad \Gamma^\chi_{\phi\phi} = -\sin\chi\cos\chi\sin^2\theta$$

$$\Gamma^\theta_{r\theta} = \Gamma^\theta_{\theta r} = \frac{1}{r}, \qquad \Gamma^\theta_{\chi\theta} = \Gamma^\theta_{\theta\chi} = \cot\chi, \qquad \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta$$

$$\Gamma^\phi_{r\phi} = \Gamma^\phi_{\phi r} = \frac{1}{r}, \qquad \Gamma^\phi_{\chi\phi} = \Gamma^\phi_{\phi\chi} = \cot\chi, \qquad \Gamma^\phi_{\theta\phi} = \Gamma^\phi_{\phi\theta} = \cot\theta$$

$$\Gamma^0_{r0} = \Gamma^0_{0r} = \frac{1}{2B}\frac{dB}{dr}$$

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